Comments Concerning Calibration

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May 24, 2001

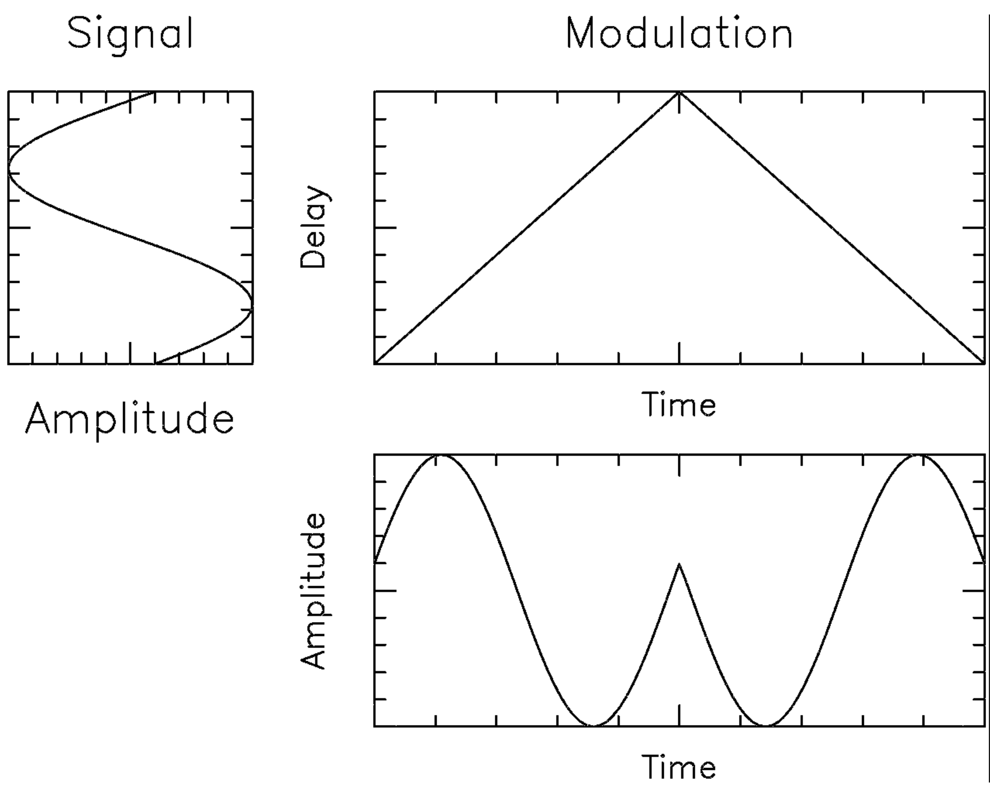
Outline of Talk

- The Fringe Detection Process
- Improving Signal to Noise
- The Need for Calibration
    - Instrumental
    - Atmosphere
- Calibration Philosophy
    - Reference Star
    - Calibration Function
    - Intrinsic Calibration

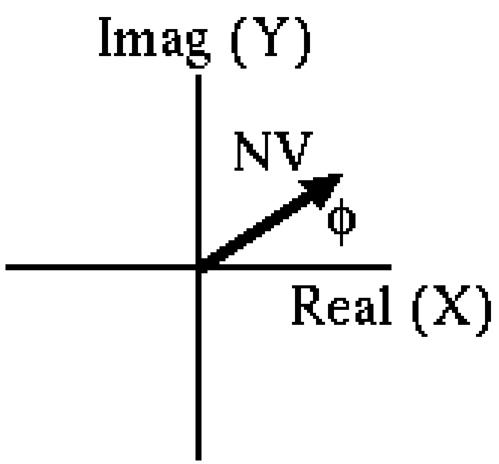
Fringe Detection – I

- Signal is Sine Wave plus noise
    - Measure Amplitude, Phase and DC Offset
- Data is digitized Signal – BINS
- Fourier Transform of Bins
  - $X + iY = Ve^{i\phi}$
- For a pupil plane system, an independent fringe is formed at each
    - Wavelength
    - Pupil position
    - time

Fringe Detection – II



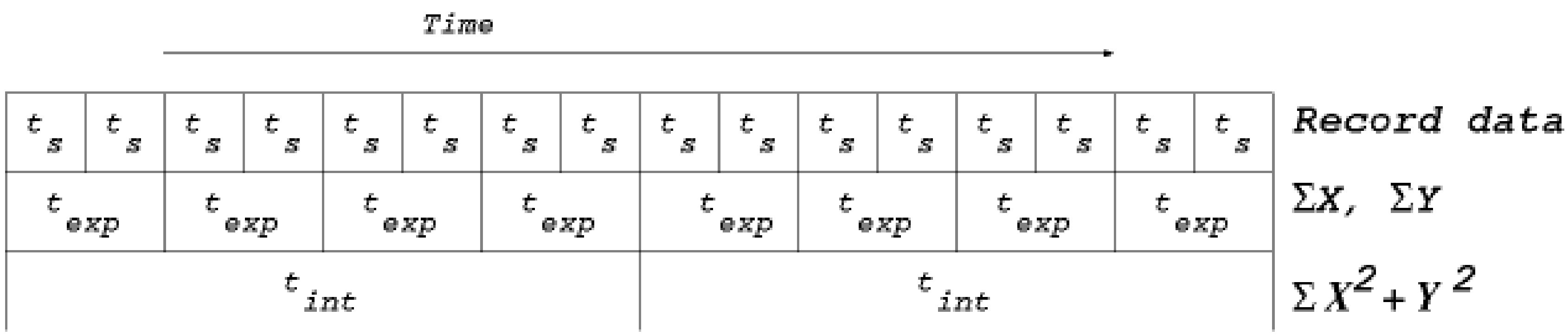
Fourier Transform



Definitions

- Sample time
  - Rate at which data is recorded
- Exposure Time
  - Period over which samples are blindly accumulated
- Integration Time
  - Time to produce one output value
- Coherent Integration
  - One which gives a Phase as well as Amplitude
- Incoherent Integration
  - One which throws away all phase information and outputs a Power (Squared Amplitude)

Example

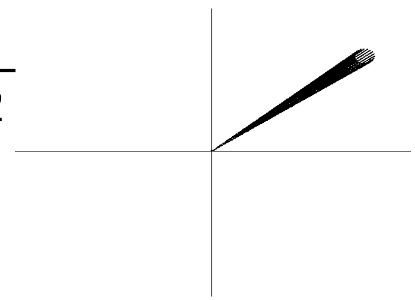


$$t_{sample} \leq t_{exposure} \leq t_{coherent} \leq t_{incoherent}$$

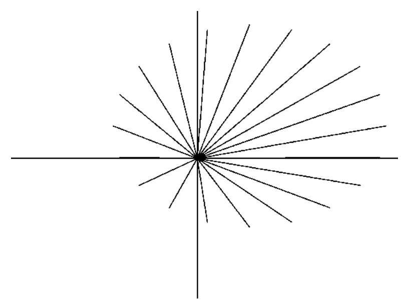
Incoherent Average

$$V_{est}^2 = \langle X^2 + Y^2 - N \rangle / \langle N \rangle^2$$

- $NV^2 >> 1 \rightarrow SNR = \sqrt{NV^2}$



- $NV^2 << 1 \rightarrow SNR = NV^2$



– Small NV^2 is bad

Fringe Detection SNR

- For SNR > 1 data, Coherent and Incoherent Integrations have the Same SNR.
- For SNR < 1 data, Coherent Integration Improves the SNR.
- A Long Exposure is a *bad* Coherent Integration since Fringe Motion during the Integration Reduces Fringe Contrast.

Coherent Integration – I

- Measure ($X(t)+iY(t)$) for each Exposure.
- Estimate Change in Phase $\Delta\phi(t)$ vs Time.
  - Wide-band fringe tracking
  - Longer wavelengths
  - Shorter baselines
- A Well-behaved Coherent Average is

$$\langle [X(t)+iY(t)]e^{-i\Delta\phi(t)} \rangle$$

Coherent Integration – II

- With good phase estimates, this coherent integration preserves fringe amplitude.

- Output is Visibility *Amplitude* and *Baseline* Phase.
    - Easy to interpret data products.
    - Feeds directly into Radio Astronomy Imaging Algorithms.

- These Phases are Uncalibrated
    - Still need Self-cal

The Need for Calibration

- The Measured Fringe Amplitude is not the Intrinsic Visibility Amplitude.

- Instrumental Effects
  - Detector Statistics
  - Fringe Tracking Error

- Atmospheric Effects
  - Coherence Time Effects
  - Coherence Length Effects
  - Scintillation and Transparency

Scintillation & Transparency – I

- Unequal Signal Strength from the two Stations Composing a Baseline Decreases the Visibility Amplitude.

$$V = \frac{\sqrt{I_1 I_2}}{(I_1 + I_2)/2} V_0$$

- Definitions
    - Scintillation – Uncorrelated between Stations.
    - Transparency – Correlated between Stations.

Scintillation & Transparency – II

$$V_{est}^2 = \left(\frac{\langle X^2 + Y^2 \rangle}{\langle N \rangle^2}\right)$$

$$\langle X^2 \rangle = \langle [NV\cos(\phi)]^2 \rangle = 4\langle I_1 I_2 \rangle V_0^2 \cos^2(\phi)$$

$$\langle Y^2 \rangle = \langle [NV\sin(\phi)]^2 \rangle = 4\langle I_1 I_2 \rangle V_0^2 \sin^2(\phi)$$

$$V_{est}^2 = \left(\frac{2\sqrt{\langle I_1 I_2 \rangle}}{\langle I_1 \rangle + \langle I_2 \rangle}\right)^2 V^2$$

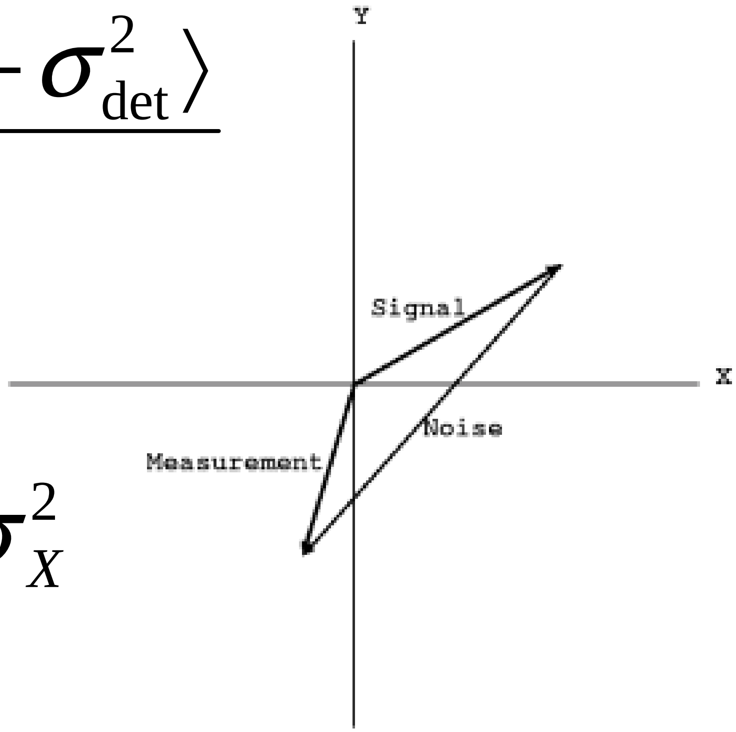
Scintillation & Transparency–III

- Intensity Fluctuations have no Effect on the Fringe Amplitude Provided

- They are Uncorrelated.
- All power is at Timescales
  - Longer than the Sample Time OR
  - Shorter than the Integration Time.

Detector Statistics – I

$$\langle V^2 \rangle = \frac{\langle X^2 + Y^2 - \sigma_{\mathrm{det}}^2 \rangle}{\langle N \rangle^2}$$



$$\langle X^2 \rangle = \langle X \rangle^2 + \sigma_X^2$$

- Measured Amplitudes are non-negative

Detector Statistics – II

- This is the *only* Effect that
  - Increases the *Measured* Fringe Amplitude.
  - Is additive, not multiplicative.
- Where σ^2 is the Detection Noise Variance.
- Photon Counters Rarely have Poisson Statistics.
- The noise Variance can be Costly to Determine and can Vary with Time.

Coping with Detectors – I

- NPOI Detectors are Photon Counting APDs

$$\sigma^2 = \sigma_0^2 + (1+\gamma)N - \mu N^2$$

  - After-pulsing – $\gamma \sim 0.05$
  - Dead Time Correction – μ
  - Temperature Dependences – μ
- All of this varies with Time
  – Must be measured during the night.

Coping with Detectors –II

- Measure σ^2 at other Fringe Frequencies.

$$z = \sigma_{est} = b_0 - b_1 + b_2 - b_3 \ldots$$

- Noise is not white
  - Exponentially distributed after-pulses
- Cross-talk
  - Modulation Frequency is Unknown (Atmosphere).
  - Side lobes of sinc Function
- Adopted Approach.

Coherence Time Effects

- Visibility Amplitude Decreases with Increasing Exposure Time due to Fringe Motion During the Exposure.

$$V = \frac{1}{\Delta t}\int_{t_1}^{t_2} \cos[(2\pi/\lambda)(d(t) - \langle d \rangle)]dt$$

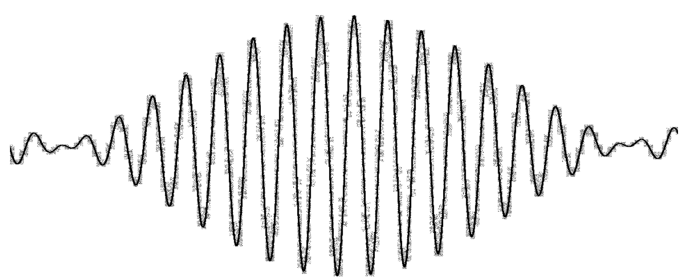
- For *Linear* Motion

$$\langle V^2 \rangle = \mathrm{sinc}^2\left(\frac{\pi \Delta d}{\lambda}\right) \approx 1 - \left(\frac{\pi^2}{3}\right)\left(\frac{\Delta d}{\lambda}\right)^2 \approx 1 - \left(\frac{1}{12}\right)(\Delta\phi)^2$$

Finite Bandwidth

- Visibility Amplitude Decreases with band pass since the Phase varies with wavelength except for the white-light fringe.

$$V = \frac{\int_{\lambda_1}^{\lambda_2} I(\lambda)\cos(2\pi d/\lambda)\,d\lambda}{\int_{\lambda_1}^{\lambda_2} I(\lambda)\,d\lambda}$$



$$V^2 \approx \mathrm{sinc}^2\left(\frac{\pi d \Delta\lambda}{\lambda^2}\right) \approx 1 - \left(\frac{\pi^2}{3}\right)\left(\frac{\Delta\lambda}{\lambda}\right)^2\left(\frac{d}{\lambda}\right)^2$$

Coherence Length Effects

- Visibility Amplitude decreases with increasing aperture size since fringe phase varies with position on the wave front.

$$V = \frac{\iint I(x,y)\cos[2\pi d(x,y)/\lambda]dxdy}{\iint I(x,y)dxdy}$$

- This is the Strehl ratio
- Current approach is to use hardware to reduce the need for this calibration

Other Amplitude Losses

- Beam Overlap
- Modulation non-linearity
- Beam Rotation
- Polarization dependent phase shifts

The Atmosphere

- Turbulence causes the wave front phase to decorrelate with both position and time.
- Coherence time, t_0, and length, r_0, are defined through their correlation functions:

$$\langle \sigma^2 (t_1 - t_2) \rangle = \left(\frac{t_1 - t_2}{t_0}\right)^{5/3}$$

$$\langle \sigma^2 (r_1 - r_2) \rangle = 6.88\left(\frac{r_1 - r_2}{r_0}\right)^{5/3}$$

Calibration Philosophy

- Reference Star
- Calibration function
- Intrinsic Calibration

Calibration Philosophy

- Usual Claim
  - Higher System Visibility is Good since it Implies Smaller Variations in the System Visibility.
- But
  - Having a Good Estimator for these Variations can be More Important.

Example

- Single Mode Fiber
  - Removes all wavefront corrugation. We must use them to improve the system visibility.
- Small Pinhole
  - works as well as a SMF.
- Large Pinhole
  - removes high order aberrations faster than tip/tilt
  - Improves the correlation between a tip-tilt measurement and the system visibility
  - Passes more Photons and may be the Better Solution.

Reference Star Approach

- Observe Stars in Pairs
  - Assume Atmosphere and Instrument are the same for both Stars in the Pair
- Divide Program star by Reference Star
- Correct for Partial Resolution of Reference
- Repeat Several Times

  Needed to Average-out Errors in Approach

Calibration Functions– I

- Observe Several Calibration Stars, widely spaced on the sky.
- Estimate System V_0 from Observations
- Fit a Function
  - Time, Zenith Angle, RA-Dec, etc
- Use function for Program Star Calibration

Calibration Function – II

- Advantages
  - Less Time Spent on Calibration
  - Self-consistency Tests are Possible
  - Needs many fewer “good” Calibrators
    - Important on Very-Long Baselines
- Disadvantages
  - Requires a well-understood, Stable Instrument.

Calibration Function – III

- Only the Mark III has Successfully used this Approach. But it may not be as Reasonable to Expect this Performance from Modern Interferometers.
  - Small Apertures
  - Short Integration Times
  - Excellent Site
  - Built Solid

Intrinsic Calibration

- Is it Possible to Calibrate Data Without Looking at Reference Stars?
- Short Answer
    - NO !

Intrinsic Calibration

- Is it Possible to Calibrate Data Without Looking at Reference Stars?
- Short Answer
  - NO !
- Longer Answer -- Maybe
  - Determine the Calibration Function from Data
  - Develop Calibration Free Data Products

Determine t_0 from Data I

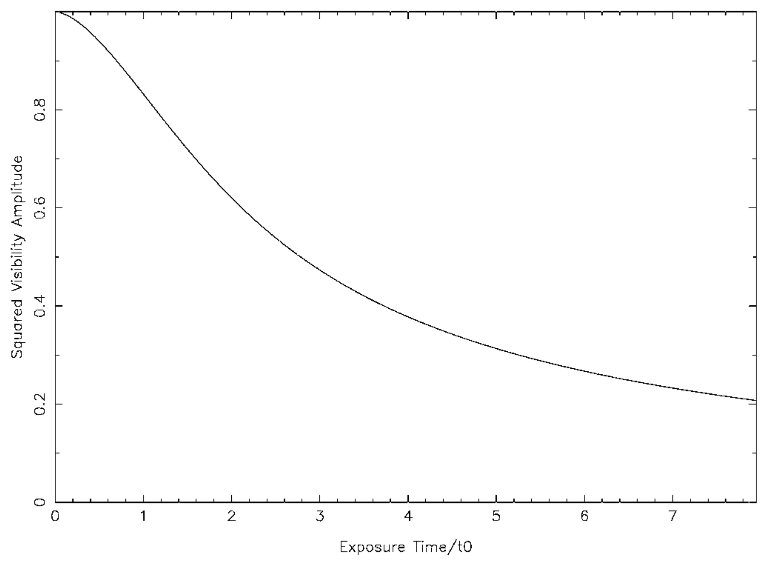
- The Visibility Amplitude decreases with increasing Exposure time.

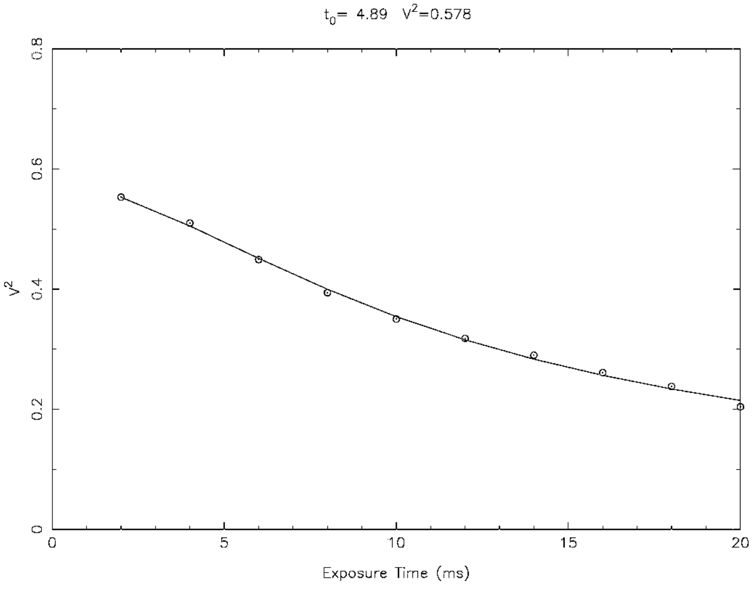
- Calculate V^2 for several exposure times.

- Extrapolate back to zero exposure time.

Determine t_0 from Data II

$$\langle V^2 \rangle = \frac{2}{t_{\exp}} \int_0^{t_{\exp}} \left(1 - \frac{t}{t_{\exp}}\right) e^{-(t/t_0)^{5/3}} dt$$





Fringe Estimation Without Detection Bias

- Make two Statistically Independent Estimates of the same (or similar) Visibility Phasor

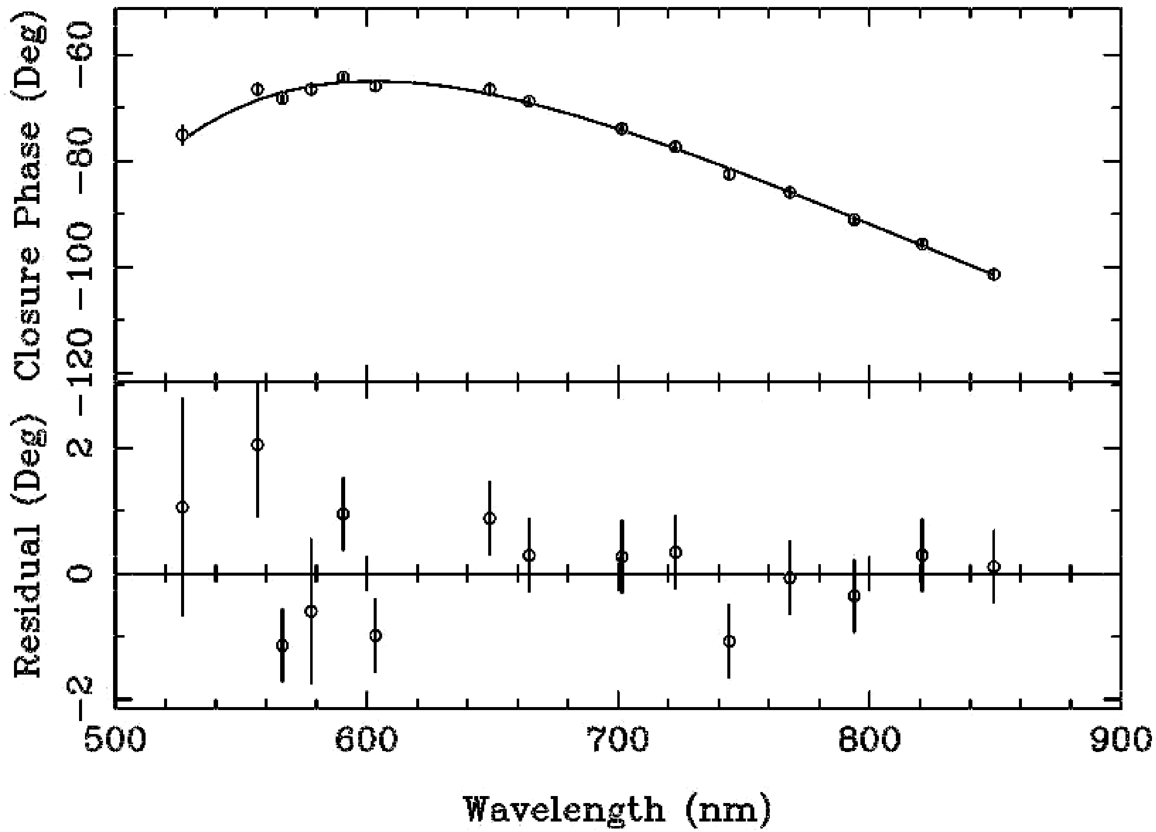
$$V_{1,2}^2 = \frac{\langle X_1 \rangle \langle X_2 \rangle + \langle Y_1 \rangle \langle Y_2 \rangle}{\langle N_1 \rangle \langle N_2 \rangle}$$

Is a V^2 Estimator without Detector noise bias.

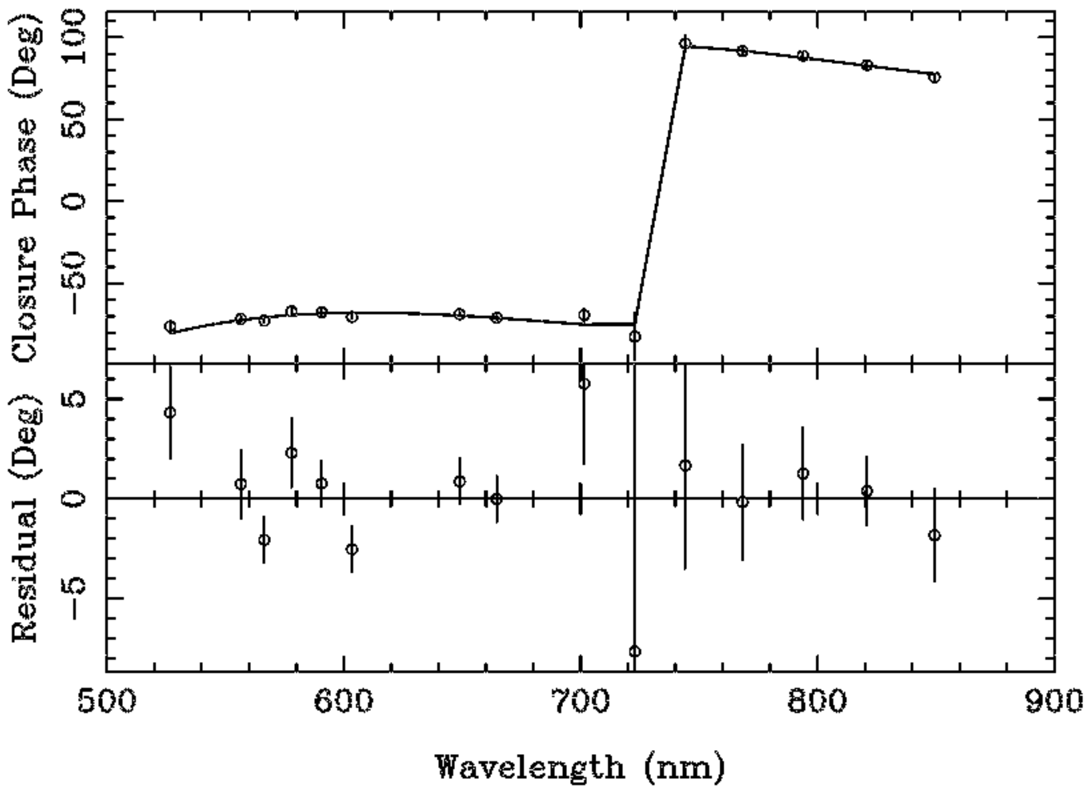
Calibration-free Data Products

- The Absolute Calibration is Noisier than its Variation with Wavelength.
- Calibration is a Smooth Function of Wavelength.
- Solve Simultaneously for
    - Calibration Parameters
    - Source Structure
- Phases –
- Amplitudes –

Phase vs Wavelength – I



Phase vs Wavelength – II



Amplitude Calibration

- Observe at Several Wavelengths
- Extrapolate V to λ where $V(\lambda)=0$
- (λ / B) is Angular-Diameter-Like.
- A Multiplicative Calibration Error does not Change λ.
- Use Estimator of V with no Detector Bias.

Closing Comments

- Understanding the Mechanisms which bias the Data is Important for Good Calibration.
    - Instrument/Data Modeling
- Reference Star Calibration is Usually Used but there are Better Approaches.
- Useful Data Products can still be Invented.
- There is Still Work to do.